Mathematics tutors' views on benefits and challenges of using Algebra tiles in teaching linear equations in one variable

ANAS SEIDU SALIFU

E.P. College of Education, Bimbilla
Ghana
salifuanas@gmail.com

ABSTRACT
This study examines the views of mathematics tutors on the benefits and challenges of using algebra tiles in solving linear equations with one variable. The qualitative research paradigm was used with a case study design. The sampling technique used was purposive with four mathematics tutors as the sample. The instrument was semi-structured open-ended interview guides. The findings revealed, among others, that the benefits of algebra tiles in solving linear equations with one variable includes (1) helping students to distinguish between coefficients and constant values, (2) the best way for teachers to explain the neutralization of equal positive and negative values. While some of the challenges of algebra tiles are (1) teachers lack the knowledge to use the algebra tiles, (2) non- availability, and inadequacy of algebra tiles.

KEYWORDS
Algebra tiles, benefits, challenges, college Mathematics teachers

RÉSUMÉ
Cette étude examine les points de vue des tuteurs en mathématiques sur les avantages et les défis de l'utilisation des tuiles d'algèbre pour résoudre des équations linéaires à une variable. Le paradigme de la recherche qualitative a été utilisé avec une conception d'étude de cas. La technique d'échantillonnage utilisée était intentionnelle avec quatre professeurs de mathématiques comme échantillon. L'instrument utilisé était un guide d'entretien semi-structuré à questions ouvertes. Les résultats ont révélé, entre autres, que les avantages des tuiles d'algèbre dans la résolution d'équations linéaires à une variable comprennent (1) l'aide aux étudiants à faire la distinction entre les coefficients et les valeurs constantes, (2) la meilleure façon pour les enseignants d'expliquer la neutralisation des valeurs égales positives et négatives. Certains des défis posés par les tuiles d'algèbre sont (1) le manque de connaissances des enseignants pour utiliser les tuiles d'algèbre, (2) la non-disponibilité et l'inadéquation des tuiles d'algèbre.

MOTS-CLÉS
Tuiles d'algèbre, avantages, défis, professeurs de mathématiques au collège

INTRODUCTION

Mathematics is one of the important subjects across the Ghanaian curriculum. The teaching and learning of it, therefore, places more emphasis on manipulative skills from the basic level to Colleges of Education and other teacher training institutions. Mathematics is perceived to be a difficult subject across the various levels of the curriculum in Ghana. Most students either fail

or score lower grades in mathematics especially at the end of their senior high schooling schedule. And that often becomes an obstacle for students to gain admission into higher institutions because of their inability to meet the minimum requirements in mathematics to progress (Ghana Education Service, 2016).

Manipulative material is seen by the Curriculum Research and Development Division (CRDD) of Ghana and National Council for Curriculum and Assessment (NaCCA) as important material or tool for teaching and learning of mathematics at all levels of education. The Ministry of Education, Transforming Teacher Education and Learning (T-TEL), and NaCCA have made it clear to all Ghanaian Basic School Teachers that they are required to include manipulative materials in preparing their lesson notes and using them for teaching (Ministry of Education, 2020). Since manipulative materials enhance understanding of concepts in mathematics during teaching and learning, the Government of Ghana in 2018 secured funding, through T-TEL Challenge Fund, for Colleges of Education to equip their resource centers with manipulatives, and to enhance teaching and learning in the Colleges. The intention of the fund is also to make these manipulatives available as a form of support to the various basic schools under various Districts, Municipalities, and Metropolis. Some Colleges also train their pre-service teachers to make or construct manipulatives for their on-campus teaching practice every year, and this automatically leads to the students submitting their project work with manipulatives used during their intervention with their pupils during the off-campus teaching practice for marks.

LITERATURE REVIEW

According to Mohd and Mohd (2010), manipulative materials can be described as small, usually, very ordinary objects that can be touched, manipulated and moved by pupils to introduce a concept or reinforce a mathematical concept during teaching. Again, Van de Walle, Karp and Bay Williams (2013, p. 24) also see manipulative material to be: "*Any object, picture, or drawing that represents a concept or onto which the relationship for that concept can be imposed. Manipulative materials are objects that pupils and teachers can use to illustrate and discover mathematical concepts, whether made specifically for mathematics (e.g., connecting cubes) or for other purposes (e.g., buttons)*".

Manipulatives are so significant that they make the teaching of mathematics easier and interesting to students and improve students' understanding of mathematical concepts and abstract ideas. Also, manipulatives enhance pupil-centered learning which improves their level of recall of previous lessons learned with ease. Manipulative is so relevant in the sense that, it introduces diversity to class activities and helps capture the students' interest which invariably increases student motivation (Cooper, 2012). Manipulative material assists in multi representational of concepts that build on students' innate of physical objects, which serves as a better foundation for abstract representations of algebraic expressions and equations in the future (Florence, 2012). Irrespective of student level, manipulatives can be used by low, average, and high achievers in class which will aid understanding of the concept. It can also engage students in the classroom for a longer period thus assisting them to concentrate on a task. Brooke (2014) study reveals that manipulatives are interactive and adaptable for teachers to use to assist students in various academic abilities. According to McIntosh (2012), "It is clear that even with minimal exposure, students of all intelligence levels can benefit greatly from the use of manipulative materials" (p. 6).

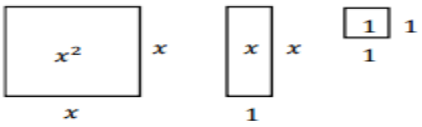
Padmore (2017) examines the use of manipulative materials in teaching Mathematics among Junior High School Teachers in the Wa Municipality of Ghana with a sample of 94 teachers and 10 head teachers. His conclusions on the benefits of manipulative use were as

follows; (1) manipulative materials improve pupils understanding and help them to construct their own knowledge of the subject easily (2) it saves a lot of time and allows teachers to cover more topics easily, motivates pupils and helps bring on board their needs to be met (3) help pupils not to shy away from mathematics but are able to relate the real-world situation to mathematical symbolism. (4) Allowing pupils to work cooperatively in solving problems, mathematics ideas and concepts and (5) make mathematics fun and easy for teachers to introduce concepts. Also, his study results on challenges of manipulative use as; (1) inadequate supply of manipulative materials to teachers (2) lack of continuous professional training on manipulative use (3) inadequate user guide for teachers on the use of manipulative materials (4) High cost in preparing and purchasing manipulative materials. (5) too much workload on teachers and (6) large class size affects the use of manipulative materials in teaching mathematics.

Boakye (2018) explored the use of mathematics manipulatives in teaching three upper public primary schools. The study used nine (9) mathematics teachers and two hundred (200) pupils as the sample. Structured questionnaires were the main instrument used to collect the data. The results of the study revealed that the use of manipulatives yielded positive results by helping teachers to clarify mathematical concepts, makes mathematics teaching very interesting and practical for the pupils. His study also reported that manipulatives help pupils to increase their performance in solving mathematical problems. Furthermore, his study indicates some challenges associated with teachers use manipulatives as; inadequacy of several manipulatives made specifically for mathematics, and difficulty in preparing or finding the right type of manipulatives to match some complex topics. According to Boakye's study, the rest of the challenges are; the use of over-complex manipulatives waste instructional time, and large class sizes making distribution and control of the use of some manipulatives difficult.

Algebra tiles are 2-dimensional rectangular shapes that are used to represent constants values and variables in mathematics. Algebra tiles use colors like yellow, blue, and green for the positive tiles, while only red represent negative tiles. Amidu, Salifu and Nyarko (2020) stated that "Algebra Tiles are a versatile manipulative that can be used by students to represent algebraic concepts beginning with integer arithmetic, continuing with activities involving linear expressions and equations, and ending with factoring and equation-solving for quadratics" p.27. The idea of the use of Algebra tiles is found in Bruner (1960), when he and his colleague Dienes wanted to teach the identity $(x+1)^2 = x^2 + 2x + 1$. Algebra tiles consist of a small square, an oblong rectangular strip, and a larger square (Figure 1).

FIGURE 1

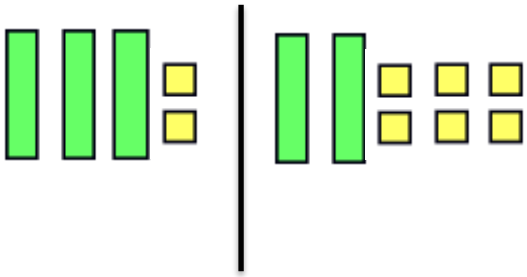


Algebra tiles unit (Picciotto & Wah, 1993)

Example of using Algebra tile approach to solve a linear equation in one variable is shown below

Example : Solve 3x+2 = 2x+6
Step 1

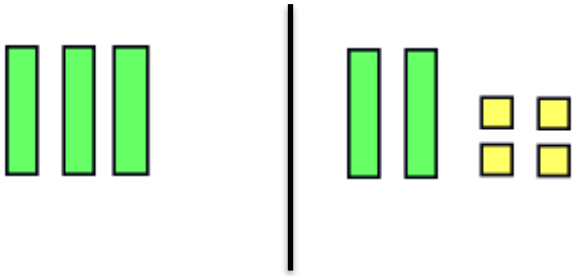
Represent 3x as 3 rectangles and 2 as 2 smaller squares as seen in the left- hand side, also represent 2x as 2 rectangles and 6 as 6 smaller squares as seen in the right- hand side.



3x+2 = 2x+6

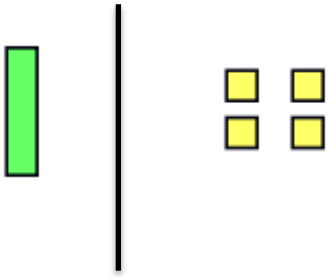
Step 2

Take away 2 smaller squares from both sides, and the remaining is 3x on the left-hand side and 2x and 4 smaller squares on the right.



Step 3

Take away 2 rectangles from both sides, and the remaining is x rectangle on the left-hand side and 4 smaller squares on the right. Therefore x= 4



Benefits of using Algebra Tiles to solve linear equations in one variable are as follows:

- Algebra tiles assists learners to make sense of mathematical problems and also enables students to explore algebraic expressions in a visual and hands-on way (Saraswati, Putri, & Somakim, 2016).
- The manipulation of the Algebra tiles can help students study the rules of algebra from their own experiences (Okpube, 2016).
- With the use of Algebra tiles, students are able to distinguish between expressions such as “2x” and “2+x” which normally confuses many students (Picciotto & Wah, 1993).
- Algebra tiles are best used to neutralized equal positive and negative values (Sibbald, 2009).
- According to Chappell and Strutchens (2001), students can use varied pairs of zero while simplifying algebraic expressions and produce several expressions without varying their values. Also, they asserted that students generally tend to use symbols such as “x” and “y” to represent variables because of the common usage of these symbols

and forget that different symbols can also be used. Algebra tiles enables learners to understand the arbitrary nature of the variable concept.

Challenges of using Algebra tiles to solve linear equations in one variable are as follows: Smith (2017) and Magruder's (2012) enumerate the following challenges associated with using Algebra tiles to Solve Linear Equations in One Variable.

- Smith (2017) states that Polynomials that exceeds first and second degree cannot be modeled with algebra tiles, besides modeling complicated examples with algebra tiles is difficult. Hence using the symbolic form for complicated examples is encouraged.
- Algebra tiles cannot denote fractions hence it is difficult to carry out the division of equations by using algebra tiles (Magruder, 2012). Furthermore, in modeling algebraic expressions with algebra tiles, one color of the rectangle algebra tile represents –x but the area cannot have a negative value in reality.

Amidu et al. (2020) revealed that there was statistically significant difference in achievement that favored the experimental group in the pre and post-test analysis. The study also indicated that the students had fun and enjoyed the usage of the algebra tiles. Furthermore, the study reported that algebra tiles helped students in understanding the concepts of algebra easily, hence they were ready to use it during on and off campus teaching practices.

Çaylan (2018) examined the effects of using algebra tiles on sixth grade students' algebra achievement, algebraic thinking and views about using algebra tiles. They employed a pretest-posttest control group design with 40 students from the sixth grade. The findings were that the algebra tiles helped the sixth-grade students to participate in the lessons actively and promoted meaningful understanding of the concepts better. The study also revealed that the participants enjoyed the lesson because they learned concepts faster and remembered concepts easily during the use of algebra tiles. The sixth-grade students also indicated that they can make a transition between concrete and symbolic representations of the concepts, explore algebraic expressions in a visual and hands-on way and learn the rules from their own experiences by the use of algebra tiles. Finally, the participants indicated that algebra tiles assisted them in avoiding mistakes and erase their confusion between expressions, and that modelling with algebra tiles enhances students' visualization skills and promotes conceptual understanding.

THEORETICAL BACKGROUND TO THIS STUDY

In the Mathematics curriculum, there exists three kinds of manipulatives, namely, concrete models, pictorial models, and symbolic models, which are used as teaching aids. The idea and use of concrete models in the teaching and learning of Mathematics has a long history that traces its source to learning theorists, Piaget and Bruner. This study is grounded in Piaget and Bruner theorems. Piaget (1964) introduced an extensive cognitive improvement theory that covers individual growth from birth to adolescence (Fennema, 1972). Applying Piaget's theory in instruction demonstrates that there should be both concrete and symbolic models in learning surroundings for children at several developmental stages. According to Piaget, children should not be restricted or play passive roles but should be allowed to interact with peers, teachers, and objects in the teaching process, which will enhance their cognitive growth. Likewise, Bruner (1961) indicated that children acquire information in three different ways. They are enactive (concrete), iconic (pictorial) and symbolic (abstract). In the enactive stage, knowledge is acquired by children via interacting with objects and concrete experiences. In the iconic stage, pictorial representations gain importance. In the symbolic stage, children get information by using symbols (Erden & Akman, 1991). Language and symbols are relevant at this stage. To

support Piaget, Bruner indicated that children's use of pictures or symbols help them to gather information only after they have experienced concrete models (McBride & Lamb, 1986). Also, (Fennema, 1972) asserted that if concrete manipulatives are introduced to students before symbolic instruction, students can apply their knowledge to new cases and comprehend abstract issues of Mathematics easily. Students can easily compose relationships between the real-world and the abstract world of Mathematics through the use of concrete models.

Hall (1998) opined that concrete materials can be helpful to the classroom teacher because they can assist the teacher in explaining operations or concepts more easily than through symbols. This will invariably help students comprehend what they have been taught. Reys (1971) also advocated the wholesale use of manipulatives because they diversify educational activities, provide practice in problem-solving, enable abstract issues to be presented concretely, enable students to participate actively, cater for individual differences, and enhance the motivation for all Mathematics topics. Similarly, Fennema (1973) argued that manipulatives increase students' motivation, arouse students' interest, and enable them to conjecture what new concepts they should learn.

Statement of the problem

Manipulatives are essential instructional tools for teaching and learning mathematics at all levels of education. With the abundant benefits of manipulatives, most teachers are still not using manipulatives in teaching mathematical concepts due to several reasons, because some of them are not sure at what stage they need to use them. Manipulatives can be used in classrooms to explain abstract concepts. Manipulatives can serve several different purposes: to introduce and develop new concepts, to present a particular problem, present abstract terms, and to be used as a problem-solving tool. Teachers have different opinions about integrating manipulative in teaching mathematics because some teachers say that it does not enhance lesson delivery in schools but others disagree with that assertion. This debate of either integrating manipulative or not is a continuous process that is worth investigating to contribute to academic knowledge. As such, there is no known research that sort to investigate the views of College Mathematics Teachers on the use of algebra tiles in teaching linear equations in one variable in Colleges of Education in Ghana. Therefore, this study examines Mathematics Tutors' Views on Benefits and Challenges of Using Algebra Tiles in teaching linear equations in one variable at Evangelical Presbyterian (E. P.) College of Education, Bimbilla, Ghana.

Significance of the study

While there is an improved pedagogy in Colleges of Education in Ghana (T-TEL, 2019), Ministry of Education (2019) is still worried about the existing gap between curriculum reforms and teachers' low usage of manipulatives in classroom. It is against this backdrop of the existence discrepancies between the curricula prescriptions and curricula realities, that this study sought to explore by narrowing the gap in the literature on teachers' views on the benefits and challenges of Algebra Tiles manipulative usage in Ghana. This study will also serve as a baseline document for future studies in teachers' views on the benefits and challenges of Algebra Tiles in teaching linear equations in one variable in Ghana.

Purpose of the study

The purpose of this study is to examine mathematics tutors' views on the benefits and challenges of using Algebra Tiles in solving linear equations with one variable in E.P. College of Education, Bimbilla.

Research questions

1. What are the views of Tutors of Mathematics in E. P. College of Education, Bimbilla on the benefits associated with Algebra Tiles manipulative use in solving linear equations with one variable?
2. What are the views of Mathematics Tutors in E. P. College of Education, Bimbilla on the challenges associated with Algebra Tiles manipulative use in solving linear equations with one variable?

METHODOLOGY

The qualitative research paradigm approach was used with semi-structured open-ended interviews. This was done with case study design, specifically in an instrumental case. The instrumental case provides insight or an in-depth understanding of the issue. The population for the study was all Mathematics Tutors of E.P. College of Education, Bimbilla in the Northern Region of Ghana. The sample for the study was 4 Tutors of Mathematics in the College, selected based on their knowledge and many years of experience in teaching Mathematics at the College in using the manipulatives. The sampling techniques used in this study were purposive and concept or theory-based. Concept or theory-based procedure was used because the participants selected for the study had rich information and are experienced in the teaching of mathematics. Purposive was also adopted because the researcher actually targeted mathematics college tutors. The instrument used in this study was a semi-structured open-ended interview developed by the researcher to collect data on the benefits and challenges of algebra tiles. The semi-structured open-ended interview was designed taking into consideration the objectives of the study. A pilot study was conducted in Bimbilla to 3 Basic School Teachers. The pilot study was done to identify possible challenges for rectification before the main study. Fortunately, no challenges were found in the pilot study, hence no revision was done.

In addressing ethical issues, consent of the Mathematics Tutors was sought through personal contact, and an assurance was given that the mathematics tutors could withdraw from the study at a time they felt uncomfortable to continue. Dates and specific times were agreed for the interviews based on the convenience and availability of each mathematics tutor. The researcher was open and honest with participants by disclosing the purpose of the research to participants. The researcher arranged an interview session with Tutors of Mathematics in the College through one-on-one interview in the tutors' homes from July 1 to 17, 2020 due to the COVID - 19 pandemic after observing their lessons earlier. The researcher role was a complete insider because the researcher is a member of the mathematics department of the College.

To ensure the validity and reliability of the semi-structured interview guide were given to experts in mathematics education for their assessment and comments on the items in the scope of clarity, ambiguity, relevance, and generality. The researcher further ensured ethical measures during the data collection process such as confidentiality, anonymity, and voluntary participation were all held in high esteem. The researcher ensured that the process of data collection was engaging in a genuine and natural environment. In the semi-structured open-ended interview, the participants were asked the same questions but in a different order. Data collection methods that were used to increase validity are a combination of participants' verbatim accounts and participant review. The researcher asked participants to review the researcher's synthesis of the interviews with participants for accuracy of representation. The participants were made to read the transcribed conversation to verify whether the data captured accurately reflected their actual position or ideas. During the interviews, audio recordings were done to serve as verbatim accounts of what transpires in the interview session material for reliability checks. Immediately following the interview, the researcher manually transcribed the

audio recordings by typing in text. This was done to remove all overlapping statements and assisted in obtaining verbatim accounts from the participants.

The responses were coded, described, and categorized according to similar patterns based on the themes of the study. Also, connections and interrelations were found in the data which facilitated the narrative discussions which corroborate the themes of the study. The verbatim presentation was done in the sense that it captured the date, and informant identity or code to ensure anonymity. The thematic analysis procedure was adopted to analyze the semi-structured open-ended interview data. The researcher first transcribed and grouped all the semi-structured open-ended interview responses into appropriate themes that match the objectives of the study. The responses were grouped and paraphrase by considering the patterns and relationships. Some responses were also written verbatim to support the themes.

RESULTS

The results of the study were presented and discussed according to the research questions posed in this study. The findings are also compared with existing literature through discussions. Research Question 1 sought to examine views of Tutors of Mathematics on the benefits associated with Algebra Tiles' manipulative use in solving linear equations with one variable in E. P. College of Education, Bimbilla. The research participants were asked to share their opinions or views on benefits associated with Algebra Tiles' manipulative use in solving linear equations with one variable. The Tutors of Mathematics in the College identified the following benefits of Algebra Tiles which were captured as the themes: distinguished between coefficients and constant values, explain the neutralized principles by adding equal positive and negative values, fun and easy way to introduce a mathematical concept, relate real-world situations to mathematics symbolism, improves easy understanding of mathematical concepts, explore algebraic expressions in a visual and hands-on way and help students study the rules of algebra from their own experiences.

On part of these themes: distinguishing between coefficients and constant values, explaining the neutralized principles of adding equal positive and negative values, fun and easy way to introduce a mathematical concept using Algebra Tiles manipulative, Tutors 1 and 2 had this to say.

A quote from respondent one conveys this view: "*I often use algebra tiles to explain the difference between coefficients and constant values when am teaching linear equations with one variable. I represent the terms such as 2x with 2 oblong rectangles while I present the constant values such as 2 with the two small squares. So with this example, I am able to explain the difference between coefficients and constant values easily without any problem. I also use this manipulative to explain the zero concepts better because it the easier to do so. For instance +4x – 4x = 0. I present 4 oblong green squares for the positive 4x and 4 small red squares for the negative 4x, then when paired they neutralize one another and nothing is left. The pre-service teachers like this procedure of explaining or working linear equations with one variable*" (Tutor 1, July 2020).

Another respondent added: "*Hmm, for me I can confidentially say that it helps me in introducing difficult mathematical concepts. For instance, the introduction of negative integers when dealing with linear equations because at the lower level pupils are taught that 2-3 answer is, it cannot be solved. Pupils were taught that when you cannot remove 3 from 2. They carry this idea to the junior high school with the notion that 2x - 3x which is negative x cannot be solved. Hence, Algebra tiles are the best manipulative for introducing negatives values in integers and linear equations with one variable. Again, these manipulative makes mathematical*

concepts easy to understand and it creates fun in the form of a game when teaching algebra" (Tutor 2, July 2020).

With regards to relating real-world situations to mathematical symbolism, and improving easy understanding of mathematical concepts, the Mathematics Tutors 1 and 4 started their views respectively as follows: "*You see algebra tiles aid mathematical symbolism which is related to real-life situations especially with regards to borrowing. For example, Mr. Shirazu has 3 mangoes which are 3x and Madam Safura borrows 2 mangoes (2x) and ate, how many mangoes will be left with Mr. Shirazu? One mango which is x will be left. Another benefit is that, algebra tiles assist in enhancing understanding linear equations in one variable because misconceptions and errors are reduced drastically in solving questions*" (Tutor 4, July 2020).

Another respondent added: "*I like this manipulative because it helps students to understand the mathematical concepts easily especially linear equations in one variable because students easily see the difference between 4x and 4. Also, instead of using abstract terms like x, I used several items such as pencils, pens, sharpeners, etc to represent x hence making the lesson more practical and real*" (Tutor 1, July 2020).

On the issue of explore algebraic expressions in a visual and hands-on way and help students study the rules of algebra from their own experiences, the Mathematics Tutors 3 and 2 also commented as follows: "*Algebra tiles are significant in the sense that it aids students to represent algebraic expressions in a visual and practical way which makes lessons more practical and real. Another benefit of this teaching and learning material is that it helps to study the rules of algebra. For instance, adding or subtracting both sides of an equation using tiles is easy to do*" (Tutor 3, July 2020).

Another respondent narrated that: "*With Algebra tiles, students are able to look at the rules of algebra from their own point of view and experience and apply the rules logically*" (Tutor 2, July 2020).

Research Question 2 sought to examine views of Tutors of Mathematics on challenges associated with Algebra Tiles manipulative use in solving linear equations with one variable in E.P. College of Education, Bimbilla. The research participants were asked to share their opinions or views challenges associated with Algebra Tiles manipulative use in solving linear equations with one variable. The Mathematics College Tutors' identified the following as challenges associated with Algebra Tiles manipulative use in solving linear equations with one variable which were captured as the themes: Polynomials that exceeds first and second degree cannot be modeled, difficult to carry out the division of equations, cannot denote fractions, lack of requisite knowledge to use Algebra Tiles manipulative, Non- availability and inadequacy of manipulative, lack of continuous professional training, lack of adequate user guides, high cost of preparing and purchasing, Workload on teachers, large class size and time for mathematics lesson is too short.

To confirm the polynomials that exceed first and second degree cannot be modeled, large class size and difficult to carry out the division of linear equations challenges, Tutors 1 and 2 had this to say "*Algebra tiles can be used for teaching linear and quadratic equations but its challenge is that it cannot be used to teach cubic equations or polynomials beyond the second degree, is difficult and complex to model. For me using Algebra tiles to teach linear equations at a higher level will be more confusing because by then students will be able to solve linear equations. Another challenge I encountered with algebra tiles is that I cannot divide tiles when teaching linear equations because for instance 2x= 5, will give x= 2.5 which cannot be represented by a tile*" (Tutor 1, July 2020).

Another respondent narrated that: "*Hmmm, in a situation where the class size is large like we have in this College, tutors find it difficult to manage teaching with the algebra tiles with a huge number of students. Another challenge with the Algebra tiles model is that it is suitable with first-degree degree polynomial, e.g. 2x + 1 = 7 but if polynomial exceeds first degree, it*

will be very difficult to deal with. For example, $2x^2 + 4x + 6 = 0$ *and* $4x^3 + 4x^2 + 4x + 4 = 0$" (Tutor 2, July 2020).

To corroborate the theme, Tutors 1 and 4 had this to say about the lack of knowledge to use of manipulative, non- availability and inadequacy of manipulative, lack of continuous professional training challenges. The following comments point to the themes: "*For me, teachers using algebra tiles need to be trained and continuously undergo training because there are some examples that are challenging, and to overcome this challenge, teachers need to always revise before coming to class. Another challenge is that as a teacher I cannot provide algebra tiles for every student before teaching because the time allocated to mathematics methods or content lessons is inadequate to continue the usage of the manipulative*" (Tutor 1, July 2020), "*……Let me use myself as an example when I first came to this college and was teaching linear equation with one variable in my first semester I never use algebra tiles. Even though, it was specifically mentioned in the syllabus because I did not have any knowledge about it. Hence lack of knowledge about its use is a major challenge. If not because of the new curriculum where teachers meet every week and plan lessons together I would not have learned how to use algebra tiles. In summary lack of continuous professional training is another big challenge*" (Tutor 4, July 2020).

On the part of lack of adequate user guides, and workload on teachers' challenges, Tutors 2 and 4 narrated as follows; Respondent 2 for instance, disclosed that: "*The period allocated for mathematics lessons in this college is so short that you need more time to be able to have a successful lesson, simply put the work schedule or pressure on we the mathematics teachers to complete syllabus is another major challenge. Hence, I sometimes find it difficult in using algebra tiles because it will slow down my speed in lesson delivery*" (Tutor 2, July 2020).

A similar view was conveyed by respondent 4: "*See it is funny, in my college days our tutor just mentioned algebra tiles as a teaching-learning material without teaching us how to use it. If we were taught or there was a user guide for it I would have read it and begin using it in this college a long time ago.so for my lack of user manual for algebra tiles is a serious hindrance, I only learned how to use it by watching online videos*" (Tutor 4, July 2020).

DISCUSSIONS OF RESULTS

This section presents the findings of the study which is compared with existing literature on the various themes. In relation to the research question one, the findings from the interview clearly point to the fact that the benefits of using algebra tiles help teachers to explain the concepts of coefficients and constant easily, because the algebra tiles for the coefficients and the constant values are different. Following this, there is no way a student will get confused with the presentation of the variables in their future teaching of linear equations in one variable. This finding conforms to Amidu et al. (2020) who also discovered that algebra tiles help students to understand algebra concepts easily and were ready to use it in the future during on and off campus teaching practices when teaching linear equations. Algebra tiles are seen as the best way for teachers to explain the neutralized principles by using negative and positive tiles to neutralize one another because there are various ways you can add or subtract algebra tiles to arrive at zero which agrees with (Sibbald, 2009) who opine that the algebra tiles is best use in explaining the neutralized principles where equal positive and negative values are added together. For instance, (i) using adding a negative bigger tile to a positive bigger tile will neutralize one another to give you zero and vice versa, signifying that $-x^2 + x^2 = 0$ *or* $x^2 - x^2 = 0$. (ii) Adding 2 negative oblong tiles to 2 positive oblong tiles will neutralize one another which give zero as the answer that is $-2x + 2x = 0$ *or* $2x - 2x = 0$. (iii) Adding negative smaller tiles to a positive smaller tiles will neutralize one another which give zero as the answer

that is $4-4=0, -5+5=0$ and so on. The algebra tiles usage creates fun and made it easy to introduce difficult mathematical concepts. For instance, the tiles help teachers to introduce concepts like $-3+(-3)=-6$, $-3-(-3)=0$, $6+(-2)=4$, $-(-6)-2=4$. Algebra tiles also helps in relating mathematical concepts to real-world situations which improve pre-service teachers' understanding and rules governing linear equations in one variable. For example, it helps teachers to explain and plan activities that enhance the understanding of linear equations rules in a more practical fashion by adding or removing algebra tiles from both sides which promote quick comprehension of linear equations rules. The adding or removing algebra tiles from both sides also helps students to explore algebraic expressions in a visual and hands-on perspective. This finding is in consonance with Saraswati et al. (2016) who study found that algebra tiles help students to explore algebraic expression in a visual and hands-on way. This study also tallies with that of Okpube (2016) who indicated that algebra tiles help students to study or learn rules of the algebra. This study further supports the studies of Picciotto & Wah (1993) and Sibbald (2007) who indicated that algebra tiles help students to distinguish between constant values and coefficients, and neutralized principles by adding equal positive and negative values respectively.

For research question two, the findings from the interview clearly points to the challenges of using algebra tiles. These challenges include teachers' lack of knowledge to use the algebra tiles manipulative because of the unavailability of user guides on algebra tiles in their teaching of linear equations in one variable. If there were user guides for manipulative materials like algebra tiles, teachers who never encountered it would have read it and try to teach their students using algebra tiles. These teachers could also organize workshops for their colleagues which will invariably enhance teachers' knowledge of using algebra tiles in teaching. This finding is similar to Padmore's (2017) study where he indicated that lack of knowledge and user guides are challenges in using algebra tiles manipulative in teaching linear equations in one variable.

Also, non- availability and inadequacy of algebra tiles is seen as a challenge to college mathematics tutors because some teachers have the knowledge in using algebra tiles in teaching but the algebra tiles are not in the schools or are in limited quantity for them to use in their classrooms. In another development lack of continuous professional training on algebra tiles for fresh tutors in teaching linear equations in one variable is another challenge as stated by the tutors, because some pre-service teachers even complete college without even knowing what algebra tiles are. So when they are posted in their schools without any professional training on that they continue to be ignorant about it even though it is in the syllabus they keep dodging or skipping it usage in classroom. Furthermore, it is difficult to use algebra tiles because of the workload since it uses consumes a lot of time and finally, teachers find it very difficult in using algebra tiles to carry out the division of equations. Specifically, dividing constant values by one another where the numerator is less than the denominator e.g $4x = 2$, $x = 1/2$. The finding of this study also agrees with the findings of (Magruder, 2012) who also found that algebra tiles cannot denote fractions when dividing equations.

CONCLUSION

This study explored mathematics tutors' views on the benefits and challenges of using algebra tiles in solving linear equations with one variable in E. P. College of Education, Bimbilla. From the findings of this study, it is concluded that algebra tiles are good manipulatives in solving linear equations in one variable because their benefits help in improving understanding of difficult mathematical concepts and help in relating mathematics concepts to real-life situational context.

RECOMMENDATION

Based on the findings of this study, the following recommendations are worthy suggestions to help address the challenges of algebra tiles so as to ensure improvement in PSTs performance in linear equations with one variable at E.P. College of Education, Bimbilla.

1. Lack of knowledge of the use of algebra tiles could be solved through workshops or continuous professional training organized by mathematics departments of colleges of education with development and sharing user guides of manipulatives.
2. Non- availability and inadequacy of algebra tiles can be addressed where college tutors train PSTs to construct algebra tiles in groups every year.
3. To address the high cost of preparing and purchasing algebra tiles local or low-cost materials should be used to improvised.

LIMITATIONS OF THE STUDY

In conducting this study, some limitations were encountered by the researcher. First, the sample size of four (4) was too small, all the mathematics tutors of the college should have been used for the study. Hence, the small number of mathematics tutors used meant that readers of this article should be cautious in generalizing this study findings for the population of the Mathematics Tutors of E. P. College of Education, Bimbilla. A larger sample may have produced different or additional themes for this study. Another limitation of the study may be the inexperience of the researcher on how to conduct interview.

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